1.1 Background

The angular speed of the SRV02 load shaft with respect to the input motor voltage can be described by the following first-order transfer function

$$\frac{\Omega_l(s)}{V_m(s)} = \frac{K}{(\tau s + 1)} \tag{1.1.1}$$

where $\Omega_l(s)$ is the Laplace transform of the load shaft speed $\omega_l(t)$, $V_m(s)$ is the Laplace transform of motor input voltage $v_m(t)$, K is the steady-state gain, τ is the time constant, and s is the Laplace operator.

The SRV02 transfer function model is derived analytically in Section 1.1.1 and its K and τ parameters are evaluated. These are known as the nominal model parameter values. The model parameters can also be found experimentally. Sections 1.1.2.1 and 1.1.2.2 describe how to use the frequency response and bump-test methods to find K and τ. These methods are useful when the dynamics of a system are not known, for example in a more complex system. After the lab experiments, the experimental model parameters are compared with the nominal values.

1.1.1 Modeling Using First-Principles

1.1.1.1 Electrical Equations

The DC motor armature circuit schematic and gear train is illustrated in Figure 1.1. As specified in [6], recall that R_m is the motor resistance, L_m is the inductance, and k_m is the back-emf constant.

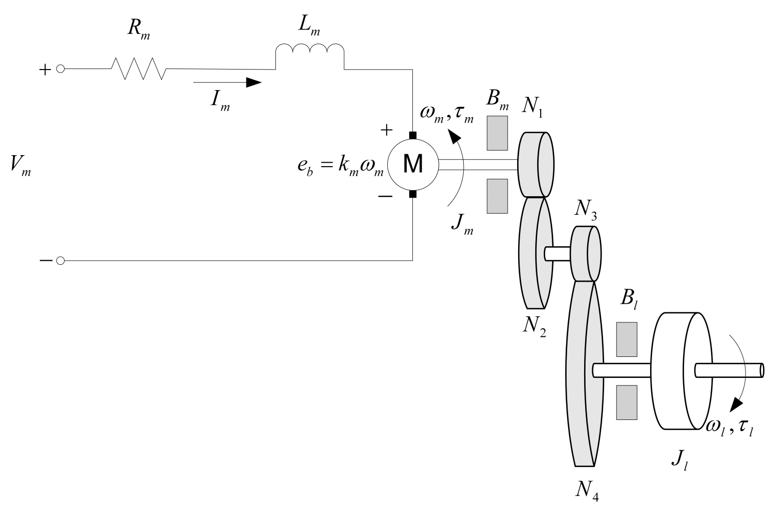

Figure 1.1: SRV02 DC motor armature circuit and gear train

The back-emf (electromotive) voltage $e_b(t)$ depends on the speed of the motor shaft, ω_m, and the back-emf constant of the motor, k_m. It opposes the current flow. The back emf voltage is given by:

$$e_b(t) = k_m \omega_m(t) \tag{1.1.2}$$

Using Kirchoff's Voltage Law, we can write the following equation:

$$V_m(t) - R_m I_m(t) - L_m \frac{dI_m(t)}{dt} - k_m \omega_m(t) = 0 \tag{1.1.3}$$

Since the motor inductance L_m is much less than its resistance, it can be ignored. Then, the equation becomes

$$V_m(t) - R_m I_m(t) - k_m \omega_m(t) = 0 \tag{1.1.4}$$

Solving for $I_m(t)$, the motor current can be found as:

$$I_m(t) = \frac{V_m(t) - k_m\omega_m(t)}{R_m} \tag{1.1.5}$$

1.1.1.2 Mechanical Equations

In this section the equation of motion describing the speed of the load shaft, ω_l, with respect to the applied motor torque, τ_m, is developed.

Since the SRV02 is a one degree-of-freedom rotary system, Newton's Second Law of Motion can be written as:

$$J \cdot \alpha = \tau \tag{1.1.6}$$

where J is the moment of inertia of the body (about its center of mass), α is the angular acceleration of the system, and τ is the sum of the torques being applied to the body. As illustrated in Figure 1.1, the SRV02 gear train along with the viscous friction acting on the motor shaft, B_m, and the load shaft B_l are considered. The load equation of motion is

$$J_l\frac{d\omega_l(t)}{dt} + B_l\omega_l(t) = \tau_l(t) \tag{1.1.7}$$

where J_l is the moment of inertia of the load and τ_l is the total torque applied on the load. The load inertia includes the inertia from the gear train and from any external loads attached, e.g. disc or bar. The motor shaft equation is expressed as:

$$J_m\frac{d\omega_m(t)}{dt} + B_m\omega_m(t) + \tau_{ml}(t) = \tau_m(t) \tag{1.1.8}$$

where J_m is the motor shaft moment of inertia and τ_{ml} is the resulting torque acting on the motor shaft from the load torque. The torque at the load shaft from an applied motor torque can be written as:

$$\tau_l(t) = \eta_g K_g \tau_{ml}(t) \tag{1.1.9}$$

where K_g is the gear ratio and η_g is the gearbox efficiency. The planetary gearbox that is directly mounted on the SRV02 motor (see [6] for more details) is represented by the N_1 and N_2 gears in Figure 1.1 and has a gear ratio of

$$K_{gi} = \frac{N_2}{N_1} \tag{1.1.10}$$

This is the *internal* gear box ratio. The motor gear N_3 and the load gear N_4 are directly meshed together and are visible from the outside. These gears comprise the *external* gear box which has an associated gear ratio of

$$K_{ge} = \frac{N_4}{N_3} \tag{1.1.11}$$

The gear ratio of the SRV02 gear train is then given by:

$$K_g = K_{ge}K_{gi} \tag{1.1.12}$$

Thus, the torque seen at the motor shaft through the gears can be expressed as:

$$\tau_{ml}(t) = \frac{\tau_l(t)}{\eta_g K_g} \tag{1.1.13}$$

Intuitively, the motor shaft must rotate K_g times for the output shaft to rotate one revolution.

$$\theta_m(t) = K_g\theta_l(t) \tag{1.1.14}$$

We can find the relationship between the angular speed of the motor shaft, ω_m, and the angular speed of the load shaft, ω_l by taking the time derivative:

$$\omega_m(t) = K_g \omega_l(t) \tag{1.1.15}$$

To find the differential equation that describes the motion of the load shaft with respect to an applied motor torque substitute (1.1.13), (1.1.15) and (1.1.7) into (1.1.8) to get the following:

$$J_m K_g \frac{d\omega_l(t)}{dt} + B_m K_g \omega_l(t) + \frac{J_l(\frac{d\omega_l(t)}{dt}) + B_l \omega_l(t)}{\eta_g K_g} = \tau_m(t) \tag{1.1.16}$$

Collecting the coefficients in terms of the load shaft velocity and acceleration gives

$$(\eta_g K_g^2 J_m + J_l) \frac{d\omega_l(t)}{dt} + (\eta_g K_g^2 B_m + B_l)\omega_l(t) = \eta_g K_g \tau_m(t) \tag{1.1.17}$$

Defining the following terms:

$$J_{eq} = \eta_g K_g^2 J_m + J_l \tag{1.1.18}$$

$$B_{eq} = \eta_g K_g^2 B_m + B_l \tag{1.1.19}$$

simplifies the equation as:

$$J_{eq} \frac{d\omega_l(t)}{dt} + B_{eq} \omega_l(t) = \eta_g K_g \tau_m(t) \tag{1.1.20}$$

1.1.1.3 Combining the Electrical and Mechanical Equations

In this section the electrical equation derived in Section 1.1.1.1 and the mechanical equation found in Section 1.1.1.2 are brought together to get an expression that represents the load shaft speed in terms of the applied motor voltage.

The motor torque is proportional to the voltage applied and is described as

$$\tau_m(t) = \eta_m k_t I_m(t) \tag{1.1.21}$$

where k_t is the current-torque constant $(N.m/A)$, η_m is the motor efficiency, and I_m is the armature current. See [6] for more details on the SRV02 motor specifications.

We can express the motor torque with respect to the input voltage $V_m(t)$ and load shaft speed $\omega_l(t)$ by substituting the motor armature current given by equation 1.1.5 in Section 1.1.1.1, into the current-torque relationship given in equation 1.1.21:

$$\tau_m(t) = \frac{\eta_m k_t \left(V_m(t) - k_m \omega_m(t)\right)}{R_m} \tag{1.1.22}$$

To express this in terms of V_m and ω_l, insert the motor-load shaft speed equation 1.1.15, into 1.1.21 to get:

$$\tau_m(t) = \frac{\eta_m k_t \left(V_m(t) - k_m K_g \omega_l(t)\right)}{R_m} \tag{1.1.23}$$

If we substitute (1.1.23) into (1.1.20), we get:

$$J_{eq}\left(\frac{d}{dt} w_l(t)\right) + B_{eq} w_l(t) = \frac{\eta_g K_g \eta_m k_t \left(V_m(t) - k_m K_g \omega_l(t)\right)}{R_m} \tag{1.1.24}$$

After collecting the terms, the equation becomes

$$\left(\frac{d}{dt} w_l(t)\right) J_{eq} + \left(\frac{k_m \eta_g K_g^2 \eta_m k_t}{R_m} + B_{eq}\right) \omega_l(t) = \frac{\eta_g K_g \eta_m k_t V_m(t)}{R_m} \tag{1.1.25}$$

This equation can be re-written as:

$$\left(\frac{d}{dt}w_l(t)\right) J_{eq} + B_{eq,v}\omega_l(t) = A_m V_m(t) \tag{1.1.26}$$

where the equivalent damping term is given by:

$$B_{eq,v} = \frac{\eta_g K_g^2 \eta_m k_t k_m + B_{eq} R_m}{R_m} \tag{1.1.27}$$

and the actuator gain equals

$$A_m = \frac{\eta_g K_g \eta_m k_t}{R_m} \tag{1.1.28}$$

1.1.2 Modeling Using Experiments

In Section 1.1.1 you learned how the system model can be derived from the first-principles. A linear model of a system can also be determined purely experimentally. The main idea is to experimentally observe how a system reacts to different inputs and change structure and parameters of a model until a reasonable fit is obtained. The inputs can be chosen in many different ways and there are a large variety of methods. In Sections 1.1.2.1 and 1.1.2.2, two methods of modeling the SRV02 are outlined: (1) frequency response and, (2) bump test.

1.1.2.1 Frequency Response

In Figure 1.2, the response of a typical first-order time-invariant system to a sine wave input is shown. As it can be seen from the figure, the input signal (u) is a sine wave with a fixed amplitude and frequency. The resulting output (y) is also a sinusoid with the *same* frequency but with a different amplitude. By varying the frequency of the input sine wave and observing the resulting outputs, a Bode plot of the system can be obtained as shown in Figure 1.3.

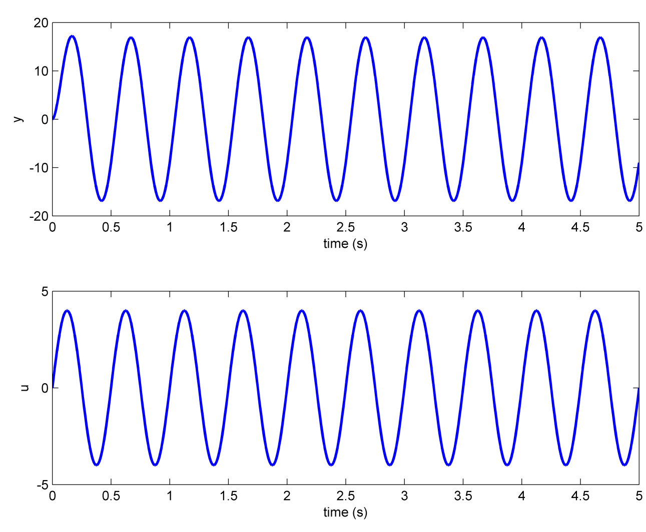

Figure 1.2: Typical frequency response

The Bode plot can then be used to find the steady-state gain, i.e. the DC gain, and the time constant of the system. The cuttoff frequency, ω_c, shown in Figure 1.3 is defined as the frequency where the gain is 3 dB less than the maximum gain (i.e. the DC gain). When working in the linear non-decibel range, the 3 dB frequency is defined as the frequency where the gain is $\frac{1}{\sqrt{2}}$, or about 0.707, of the maximum gain. The cutoff frequency is also known as the bandwidth of the system which represents how fast the system responds to a given input.