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# STUDENT WORKBOOK

## Magnetic Levitation Experiment for MATLAB®/Simulink® Users

Standardized for ABET* Evaluation Criteria

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# CONTENTS

# 1 INTRODUCTION

The MAGLEV plant is an electromagnetic suspension system acting on a solid one-inch steel ball. It mainly consists of an electromagnet, located at the upper part of the apparatus, capable of lifting from its pedestal and sustaining in free space the steel ball. Two system variables are directly measured on the MAGLEV rig and available for feedback. They are namely: the coil current and the ball distance from the electromagnet face. A more detailed description is provided in the Magnetic Levitation User Manual [4].

During the course of this experiment, you will become familiar with the design and pole placement tuning of both PI current controller and PIV-plus-feed-forward ball position controller. The challenge of the present laboratory is to levitate a one-inch solid steel ball in air from the pedestal using an electromagnet. The control system should maintain the ball stabilized in mid-air and track the ball position to a desired trajectory.

**Topics Covered**

- Modeling the MAGLEV plant from first principles in order to obtain the two open-loop transfer functions characterizing the system, in the Laplace domain.
- Linearize the obtained non-linear equation of motion about the quiescent point of operation.
- Design, through pole placement, a Proportional-plus-Integral (PI) controller for the MAGLEV electromagnet current in order for it to meet the required design specifications.
- Design, through pole placement, a Proportional-plus-Integral-plus-Velocity (PIV) controller with feed-forward action for the MAGLEV levitated ball position in order for it to meet the required design specifications.
- Implement your two controllers in real-time and evaluate their actual performances.
- Numerically determine the system's actual closed-loop poles, by considering the coil current control system's dynamics.

**Prerequisites**

In order to successfully carry out this laboratory, the user should be familiar with the following:

1. See the system requirements in Section 5 for the required hardware and software.
2. Transfer function fundamentals, e.g., obtaining a transfer function from a differential equation.
3. Familiar with designing PID controllers.
4. Basics of Simulink®.
5. Basics of QUARC®.

# 2 MODELING

## 2.1 Background

A schematic of the Magnetic Levitation (MAGLEV) plant is represented in Figure 2.1. As illustrated in Figure 2.1, the positive direction of vertical displacement is downwards, with the origin of the global Cartesian frame of coordinates on the electromagnet core flat face. Although the ball does have six Degrees Of Freedom (DOF) in free space, only the vertical, i.e., x-axis, is controlled. It can also be seen that the MAGLEV consists of two main systems: an electrical and an electro-mechanical.

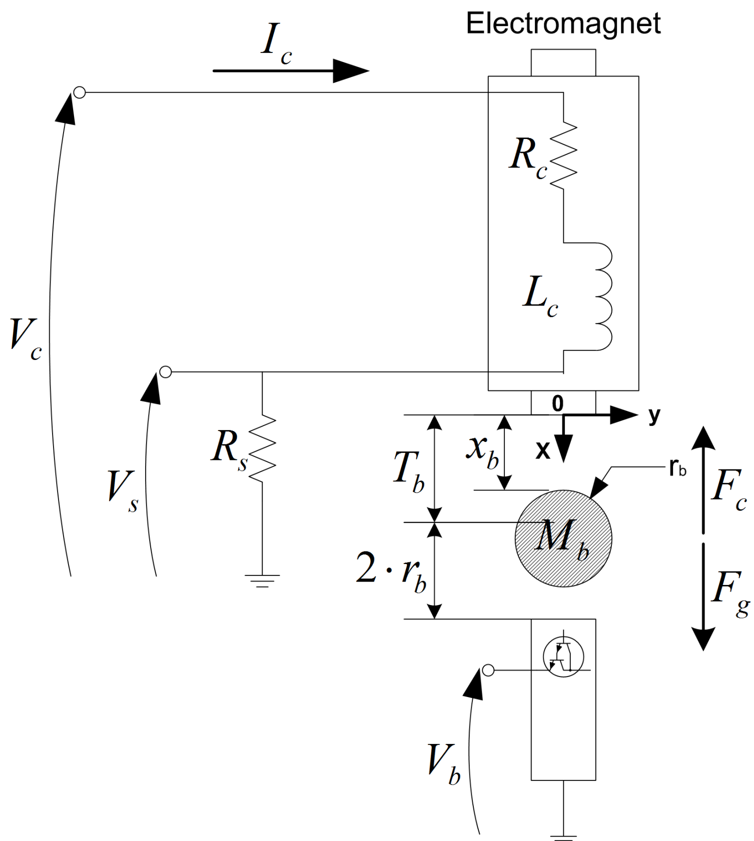



Figure 2.1: Schematic of the Magnetic Levitation plant.

### 2.1.1 Electrical Equations

As represented in Figure 2.1, the MAGLEV coil has an inductance $L_c$ and a resistance $R_c$. Additionally, the actual system is equipped with a current sense resistor, $R_s$, that is in series with the coil. The voltage sense, $V_s$, is used to measure the current in the coil. The coil current can then be computed using the following relationship

$$V_s(t) = R_s i_s(t)$$

Using Kirchhoff's voltage law, we obtain the following first-order differential equation

$$v_c(t) = (R_c + R_s)i_c(t) + L_c\frac{di_c(t)}{dt} \tag{2.1}$$

where $R_c$ is the coil resistance, $L_c$ is the coil inductance, $I_c$ is the coil current, $v_c$ is the applied coil voltage, and $R_s$ is the current sense resistance.

This can be represented by the first-order transfer function

$$G_c(s) = \frac{I_c(s)}{V_c(s)} = \frac{K_c}{\tau_c s + 1} \tag{2.2}$$

where $K_c$ is the DC (or steady-state) gain and $\tau_c$ is the time constant.

### 2.1.2 Nonlinear Model

Using the notation and conventions given in Figure 2.1, the attractive force generated by the electromagnet and acting on the steel ball can be expressed by

$$F_c = \frac{K_m i_c(t)^2}{2x_b^2} \tag{2.3}$$

where $x_b > 0$ is the air gap between the ball and the face of the electromagnet and $K_m$ is the electromagnetic force constant. The pull of the electromagnet is proportional to the square of the current and inversely proportional to the air gap (i.e., ball position) squared. The force due to gravity acting on the ball is given by

$$F_g = M_b g.$$

The total external force experienced by the ball using the electromagnet is given by

$$F_{ext} = -F_c + F_g = -\frac{K_m i_c(t)^2}{x_b(t)^2} + M_b g.$$

Applying then Newton's second law of motion to the ball gives the following nonlinear Equation Of Motion (EOM)

$$\ddot{x}_b(t) = -\frac{K_m i_c(t)^2}{2M_b\, x_b(t)^2} + g. \tag{2.4}$$

### 2.1.3 Linear Model

The nominal coil current, $i_{c0}$, for the electromagnet-ball pair can be determined at the system's static equilibrium. By definition, static equilibrium at a nominal operating point $(x_{b0}, i_{c0})$ is characterized by the ball being suspended in air at a constant position $x_{b0}$ due to a constant electromagnetic force generated by $i_{c0}$.

In order to design a linear position controller for our system, the Laplace open-loop transfer function must be derived. However, a transfer function can only represent the system's dynamics from a linear differential equation. Therefore, the nonlinear EOM given in Equation 2.4 should be linearized around a quiescent point of operation.

In the case of the levitated ball, the operating range corresponds to small departure positions, $\delta x_b$, small departure currents, $\delta i_c$, from the desired equilibrium point $(x_{b0}, i_{c0})$. Therefore, $x_b$ and $i_c$ can be expressed as the sum of two quantities, as shown below:

$$x_b = x_{b0} + \delta x_b$$

and

$$i_c = i_{c0} + \delta i_c.$$

**Example: Linearing a Two-Variable Function**
Here is an example of how to linearize a two-variable nonlinear function called $f(z)$. Variable $z$ is defined

$$z^\top = [z_1\ z_2]$$

and $f(z)$ is to be linearized about the operating point

$$z_0{}^\top = [a\ b]$$

The linearized function is

$$f_z = f(z_0) + \left(\frac{\partial f(z)}{\partial z_1}\right)\Bigg|_{z=z_0}(z_1 - a) + \left(\frac{\partial f(z)}{\partial z_2}\right)\Bigg|_{z=z_0}(z_2 - b)$$

**Open-loop Transfer Function**

The linear representation of the dynamics between the coil current and the ball position about the operating point can be represented the transfer function

$$G_b(s) = \frac{\Delta X_b(s)}{\Delta I_c(s)} = -\frac{K_b \omega_n^2}{s^2 - \omega_b^2} \tag{2.5}$$

where $\Delta X_b(s) = \mathcal{L}[\delta x_b(t)]$, $\Delta I_c(s) = \mathcal{L}[\delta i_c(t)]$, $K_b$ is the DC gain (i.e., steady-state gain), and $\omega_b$ is the natural frequency. Since we are dealing with the displacement about the operating point, the initial conditions are zero, i.e., $\delta x_b(0^-) = 0$ and $\delta i_c(0^-) = 0$.

## 2.2 Pre-Lab Questions

1. Find the open-loop electrical system transfer function that represents the coil voltage to coil current given in Equation 2.2. Assume all initial conditions are zero, i.e., $i_c(0^-) = 0$. Determine the first-order DC gain, $K_c$, and time constant, $\tau_c$, model and evaluate them numerically. See the MAGLEV User Manual [4] for system parameters.

2. Is the electrical system stable? What is its order and its type?

3. Express the static equilibrium current $i_{c0}$ as a function of the system's desired equilibrium position $x_{b0}$ and its electromagnet force constant $K_m$. Using the equilibrium position $x_{b0} = 6$ mm and the system's specifications given in MAGLEV User Manual [4], evaluate $i_{c0}$. Also, express the electromagnet force constant $K_m$ as a function of the system's desired equilibrium point $(x_{b0}, i_{c0})$.

4. Linearize the ball's EOM found in Equation 2.4 about the quiescent operating point $(x_{b0}, I_{c0})$. This will give you a function in the form $\delta\ddot{x}_b = f(x_b, i_c)$. To simplify your final equations, apply the $K_m$ expression you found in the previous exercise.

5. From the linear equation of motion, determine the system's open-loop transfer function. Express the open-loop transfer function in terms of the DC gain, $K_b$, and natural frequency, $\omega_b$. Is the system stable? What are its order and its type?

# 3 COIL CURRENT CONTROL

## 3.1 Background

### 3.1.1 Second-Order Response

The block diagram shown in Figure 3.1 is a general unity feedback system with compensator (controller) $C(s)$ and a transfer function representing the plant, $P(s)$. The measured output, $Y(s)$, is supposed to track the reference signal $R(s)$ and the tracking has to match to certain desired specifications.

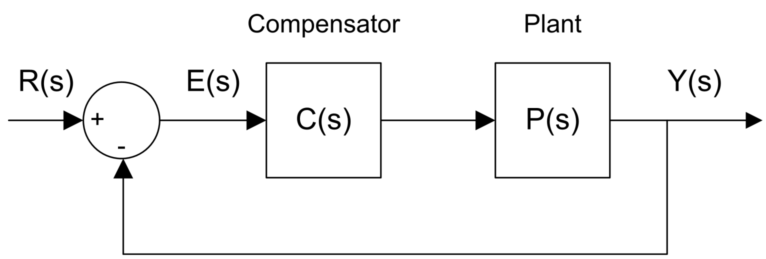



Figure 3.1: Unity feedback system.

The output of this system can be written as:

$$Y(s) = C(s)\,P(s)\,\left(R(s) - Y(s)\right)$$

By solving for $Y(s)$, we can find the closed-loop transfer function:

$$\frac{Y(s)}{R(s)} = \frac{C(s)\,P(s)}{1 + C(s)\,P(s)}$$

In fact, when a second order system is placed in series with a proportional compensator in the feedback loop as in Figure 3.1, the resulting closed-loop transfer function can be expressed as:

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\,\omega_n\,s + \omega_n^2} \tag{3.1}$$

where $\omega_n$ is the natural frequency and $\zeta$ is the damping ratio. This is called the *standard second-order* transfer function. Its response properties depend on the values of $\omega_n$ and $\zeta$.

**Peak Time and Overshoot**

Consider a second-order system as shown in Equation 3.1 subjected to a step input given by

$$R(s) = \frac{R_0}{s} \tag{3.2}$$

with a step amplitude of $R_0 = 1.5$. The system response to this input is shown in Figure 3.2, where the red trace is the response (output), $y(t)$, and the blue trace is the step input $r(t)$.

The maximum value of the response is denoted by the variable $y_{max}$ and it occurs at a time $t_{max}$. For a response similar to Figure 3.2, the percent overshoot is found using

$$PO = \frac{100\,(y_{max} - R_0)}{R_0} \tag{3.3}$$

From the initial step time, $t_0$, the time it takes for the response to reach its maximum value is

$$t_p = t_{max} - t_0 \tag{3.4}$$

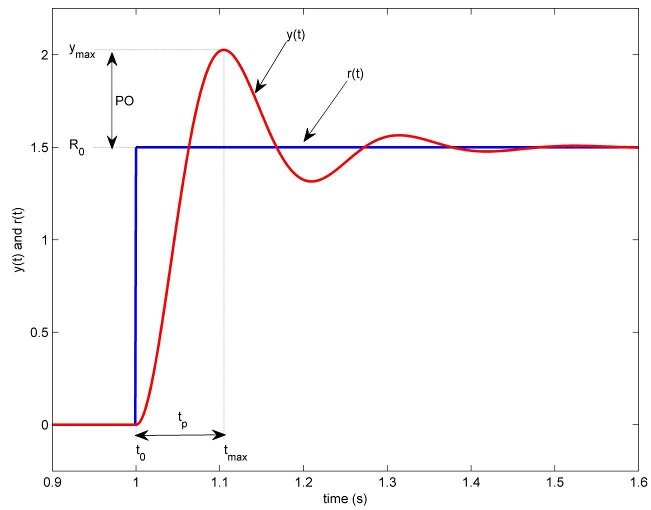



Figure 3.2: Standard second-order step response.

This is called the *peak time* of the system.

In a second-order system, the amount of overshoot depends solely on the damping ratio parameter and it can be calculated using the equation

$$PO = 100\,e^{\left(-\frac{\pi\,\zeta}{\sqrt{1-\zeta^2}}\right)} \tag{3.5}$$

The peak time depends on both the damping ratio and natural frequency of the system and it can be derived as

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \tag{3.6}$$

Generally speaking, the damping ratio affects the shape of the response while the natural frequency affects the speed of the response.

### 3.1.2 Specifications

The first closed-loop system is to control the electromagnet coil current via the commanded coil voltage. It is based on a Proportional-plus-Integral (PI) scheme. In response to a 0-to-1 A square wave coil current setpoint, tune the PI current controller in order to satisfy the following design performance requirements:

1. Maximum percent overshoot of 1.5 %, i.e., $PO_c \leq 1.5$ %.
2. No steady-state error, i.e., $e_{ss,c} = 0$.
3. Maximum peak time of 0.05 seconds, i.e., $t_{p,c} \leq 0.05$ s.

In order to obtain these requirements, you need a second-order system with the following parameters:

1. Natural frequency, $\omega_{n,c} = 104.9$ rad/s.
2. Damping ratio, $\zeta = 0.802$.

### 3.1.3 Coil Current Control Design

Prior to control the steel ball position, the current flowing through the electromagnet needs to be controlled. The electromagnet current control loop consists of a Proportional-plus-Integral (PI) closed-loop scheme, as illustrated in Figure 3.3.

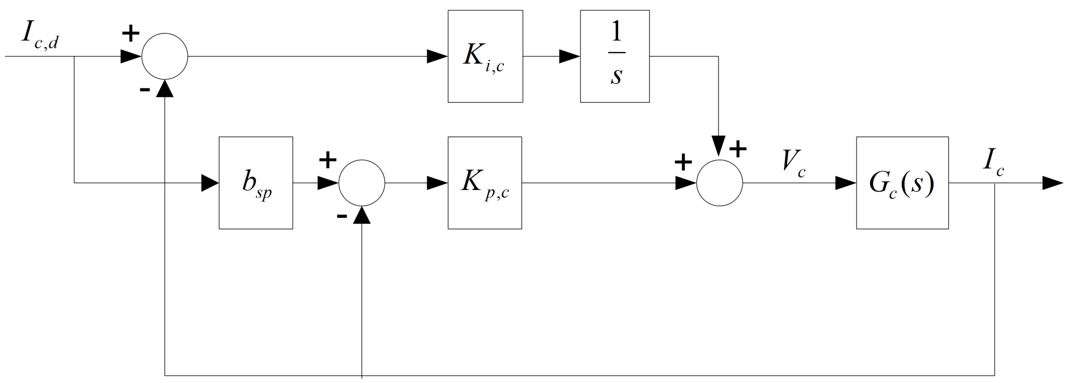



Figure 3.3: Block diagram of PI current control

The PI control has the following structure

$$V_c(s) = \left(k_{p,c} + \frac{k_{i,c}}{s}\right)(I_{c,d}(s) - I_c(s)) \tag{3.7}$$

where $k_{p,c}$ is the proportional control gain, $k_{i,c}$ is the integral control gain, $I_{c,d}$ is the desired coil current (i.e., reference or setpoint current), $I_c$ is the measured coil current, and $V_c$ is the applied coil voltage.

Substituting the PI control in 3.7 into the plant transfer function $G_c(s)$, given in Equation 2.2, and solving for $I_{c,d}(s)/I_c(s)$ gives the MAGLEV closed-loop current control transfer function:

$$T_c(s) = \frac{I_c(s)}{I_{c,d}(s)} = \frac{K_c(k_{p,c}s + k_{i,c})}{\tau_c s^2 + (K_c k_{p,c} + 1)s + K_c k_{i,c}} \tag{3.8}$$

### 3.1.4 Set-Point Weighting

The controllers described so far are called controllers with error feedback because the control action is based on the error, which is the difference between the reference $r$ and the process output $y$. There are significant advantages to have the control action depend on the reference and the process output and not just on the difference between this signals. A simple way to do this is to replace the ideal PID controller with

$$u(t) = k(b_{sp}r(t) - y(t)) + k_i \int_0^t (r(\tau) - y(\tau)d\tau - k_d \frac{dy(t)}{dt} \tag{3.9}$$

where the parameter $b_{sp}$ is called set-point weight or the reference weight. In this controller the proportional action only acts on a fraction $b_{sp}$ of the reference and there is no derivative action on the set-point. Integral action continues to act on the full error to ensure the error goes to zero in steady state.

Figure 3.4 illustrates the effects of set-point weighting on the step response of the process,

$$P(s) = \frac{1}{s}$$

with the controller gains $k_p = 1.5$ and $k_i = 1$. As shown in Figure 3.4, the overshoot for reference changes is smallest for $b_{sp} = 0$, which is the case where the reference is only introduced in the integral term, and increases with increasing $b_{sp}$. The set-point weights in Figure 3.4 are: $b_{sp} = 0$ on the bottom dashed plot trajectory, $b_{sp} = 0.2$ and $b_{sp} = 0.5$ on the two solid lines, and $b_{sp} = 1$ on the top dash-dot response. The set-point parameter is typically in the range of 0 to 1.

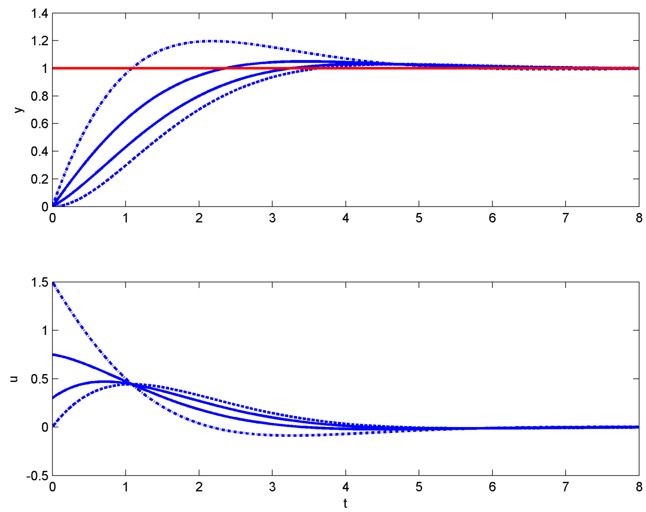



Figure 3.4: Set-point weighting effect on step response.

### 3.1.5 Integral Windup

The *windup* effect is illustrated in Figure 3.5 by the dashed red line. The initial reference signal is so large that the actuator saturates at the high limit. The integral term increases initially because the error is positive. The output reaches the reference at around time $t = 4$. However, the integrator has built-up so much energy that the actuator remains saturated. This causes the process output to keep increasing past the reference. The large integrator output that is causing the saturation will only decrease when the error has been negative for a sufficiently long time. When the time reaches $t = 6$, the control signal finally begins to decrease while the process output reaches its largest value. The controller saturates the actuator at the lower level and the phenomena is repeated. Eventually the output comes close to the reference and the actuator does not saturate. The system then behaves linearly and settles quickly. The windup effect on the process output is therefore a large overshoot and a damped oscillation where the control signal flips from one extreme to the other as in relay oscillations.

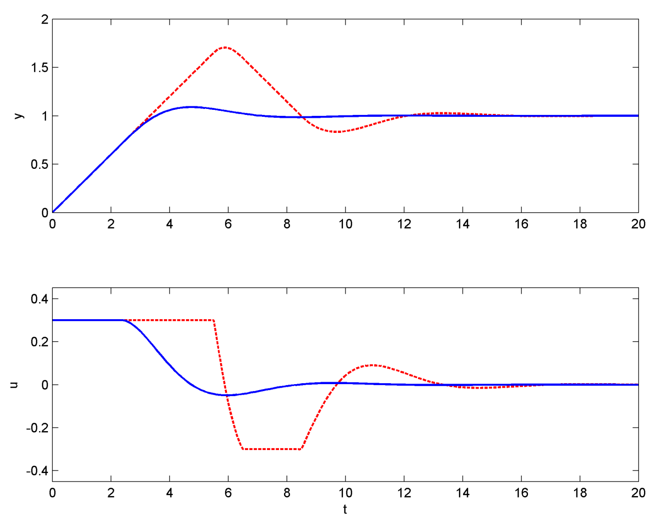



Figure 3.5: Illustration of integrator windup.

There are many ways to avoid windup, one method is illustrated in Figure 3.6 The system has an extra feedback path that that sets the integrator to a value so that the controller output is always close to the saturation limit. This is accomplished by measuring the difference es between the actual actuator output and feeding this signal to the integrator through gain $1/T_r$.

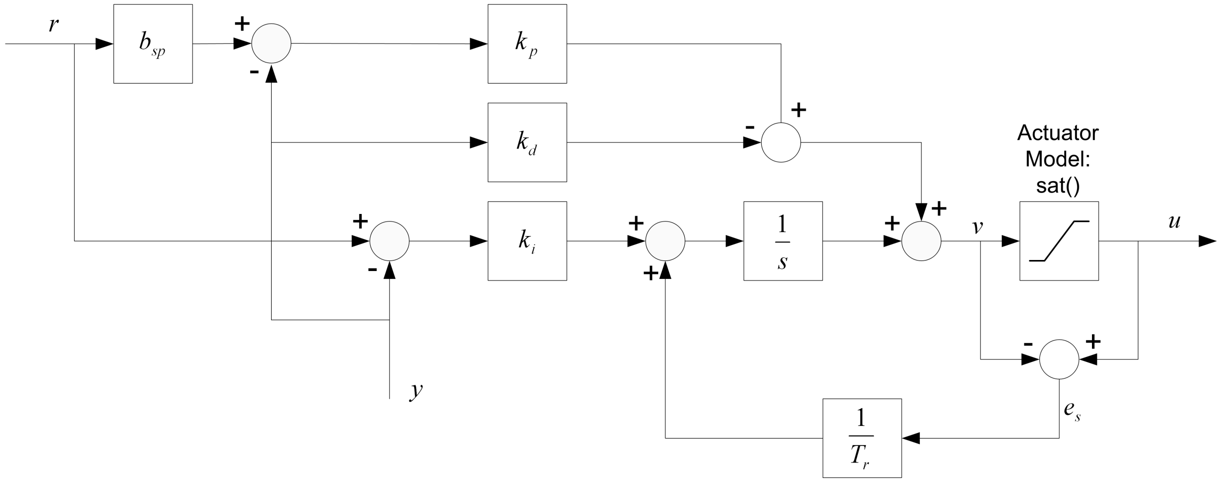



Figure 3.6: PID controller with anti-windup

The signal $e_s$ is zero when there is no saturation and the extra feedback loop has no effect on the system. When the actuator saturates, the signal $e_s$ is different from zero. The normal feedback path around the process is broken because the process input remains constant. The feedback around the integrator will act to drive $e_s$ to zero. This implies that controller output is kept close to the saturation limit and integral windup is avoided.

The solid curves in Figure 3.5 illustrates the effect of anti-windup. The output of the integrator is quickly reset to a value such that the controller output is at the saturation limit, and the integral has a negative value during the initial phase when the actuator is saturated. Observe the dramatic improvement of using windup protection over the ordinary PI controller that is represented by the dashed lines in Figure 3.5.

## 3.2 Pre-Lab Questions

1. Find the PI gains for the coil current control, $k_{p,c}$ and $k_{i,c}$, in terms of $\omega_n$ and $\zeta$. **Hint:** Remember the standard second order system equation.
2. Based on the MAGLEV model parameters, $K_c$ and $\tau_c$ found in Section 2.2, calculate the control gains needed to satisfy the time-domain response requirements given in Section 3.1.2.

## 3.3 Lab Experiments

### 3.3.1 Current Control Simulation

In this section you will simulate the magnet coil current control of the MAGLEV system. The coil is modeled using the first-order transfer function presented in Section 2.1 and controlled using the PI controller described in Section 3.1.3. Our goals are to confirm that the desired reponse specifications are satisfied and to verify that the amplifier is not saturated.

**Experimental Setup**

The *s_pi_maglev* Simulink®diagram shown in Figure 3.7 will be used to simulate the closed-loop current control response with the PI controller. The current commands are positive and limited by the rated maximum magnet coil current of 3 A.

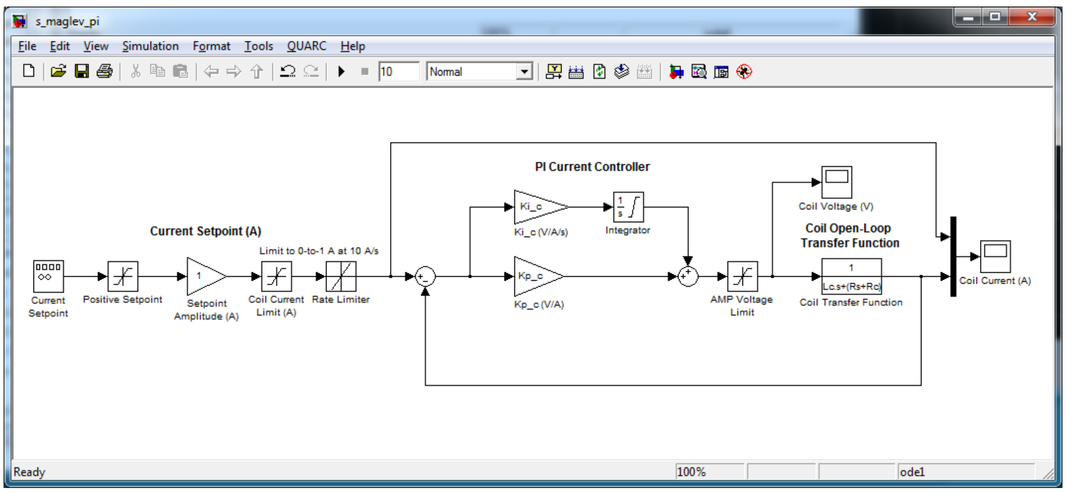
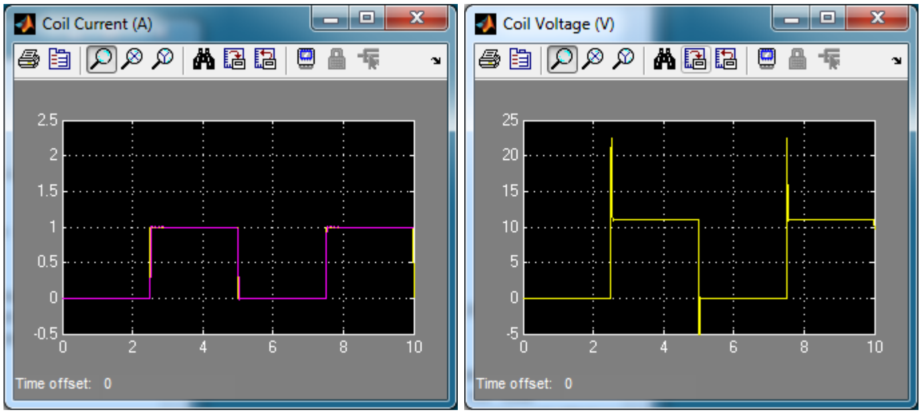



Figure 3.7: Simulink model used to simulate coil current control response.

**IMPORTANT:** Before you can conduct these experiments, you need to make sure that the lab files are configured. If they have not been configured already, then you need to go to Section 5 to configure the lab files first.

1. Enter the proportional and integral control gains found in Section 3.2 in Matlab as *Kp_c* and *Ki_c*.
2. To generate a step reference, go to the *Current Setpoint* Signal Generator block and set it to the following:
   - Signal type = *square*
   - Amplitude = 1
   - Frequency = 0.2 Hz
3. Set the *Setpoint Amplitude (A)* gain block to 1 to generate a 0 to 1 A step (i.e., square wave goes between $\pm 1$ but is passed through the Satruation block that keeps it between 0 and 1).
4. Open the *Coil Current (A)* and *Coil Voltage (V)* scopes.
5. Set the set-point weight parameter, i.e., Gain block *bsp*, to 0.
6. By default, there should be no anti-windup on the Integrator block (i.e., just use the default Integrator block).
7. Start the simulation. By default, the simulation runs for 10 seconds. The scopes should be displaying responses similar to Figure 3.8. Note that in the *Coil Current (A)* scope, the yellow trace is the setpoint position while the purple trace is the simulated current (generated by the Transfer Function block).




(a) Coil Current (b) Coil Voltage

Figure 3.8: Simulated closed-loop current control response.

8. Generate a Matlab®figure showing the *Simulated Coil Current* response and the input voltage.

   **Data Saving**: After each simulation run, each scope automatically saves their response to a variable in the Matlab®workspace. The *Coil Current (deg)* scope saves its response to the variable called data_Ic and the *Coil Voltage (V)* scope saves its data to the *data_Vc* variable.

   - The *data_Ic* variable has the following structure: *data_Ic(:,1)* is the time vector, *data_Ic(:,2)* is the setpoint, and *data_Ic(:,3)* is the simulated current.
   - For the *data_Vc* variable, *data_Vc(:,1)* is the time and *data_Vc(:,2)* is the simulated input voltage.

9. Measure the steady-state error, the percent overshoot and the peak time of the simulated response. Does the response satisfy the specifications given in Section 3.1.2? **Hint:** Use the Matlab® *ginput* command to take measurements off the figure.

10. When doing levitation control, more than 1 A is required to initially lift the metal ball from the pedestal on the MAGLEV device. To mimic the current required, simulate the system with a current setpoint step of 2 A. Examine the current and voltage responses as well as the output of the Integral in the *Integral Control (V)* scope. What is happening to the control signal and the corresponding response? How can the control be modified to address this?

11. Based on your observations in the step above and the background given in Section 3.1, modify the controller to improve the simulated closed-loop current response. Explain the control modifications you make. **Hint:** You may want to use the Matlab parameter VMAX_AMP.

12. Attach the closed-loop current response using your modified PI controller as a Matlab figure.

13. Measure the steady-state error, the percent overshoot and the peak time of the simulated response. Does the response satisfy the specifications given in Section 3.1.2? **Hint:** Use the Matlab® *ginput* command to take measurements off the figure.

## 3.3.2 Current Control Implementation

The q_maglev_pi Simulink diagram shown in Figure 3.9 is used to perform the current control exercises in this laboratory. The *MAGLEV* subsystem contains QUARC®blocks that interface with the electromagnet and sensors of the MAGLEV system.

**Experimental Setup**

The *q_maglev_pi* Simulink®diagram shown in Figure 3.9 will be used to run the PI current control on the actual MAGLEV system.

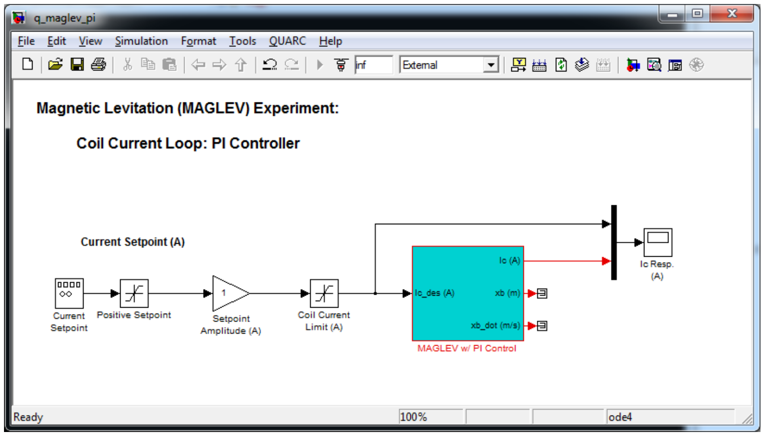
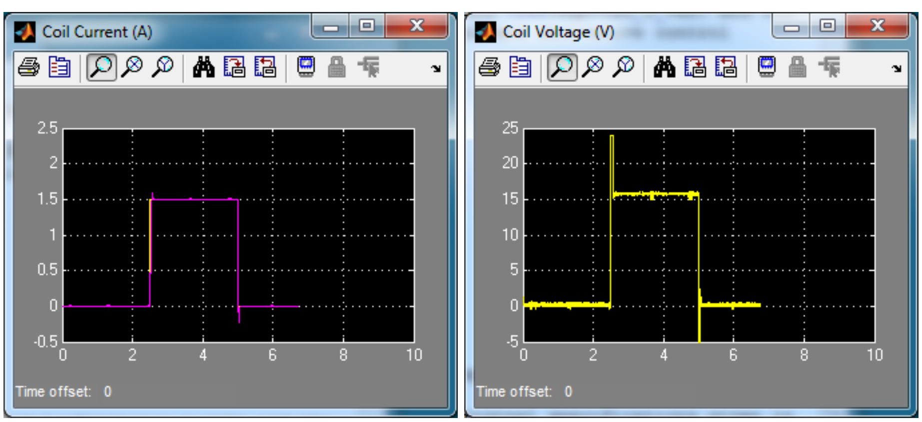



Figure 3.9: Simulink model used to run PI current control on MAGLEV system.

**IMPORTANT:** Before you can conduct these experiments, you need to make sure that the lab files are configured according to your setup. If they have not been configured already, then you need to go to Section 5 to configure the lab files first.

Follow this procedure:

1. Enter the proportional and integral control gains found in Section 3.2 in Matlab®as *Kp_c* and *Ki_c*.
2. To generate a step reference, go to the *Current Setpoint* Signal Generator block and set it to the following:
   - Signal type = *square*
   - Amplitude = 1
   - Frequency = 0.2 Hz
3. Set the *Setpoint Amplitude (A)* gain block to 1.5 to generate a 0 to 1.5 A step.
4. In the Simulink diagram, go to QUARC | Build.
5. Click on QUARC | Start to run the controller. Because we are not commanding a high current for a long period of time, the ball should not be levitating. The scopes should be displaying responses similar to Figure 3.10.




(a) Coil Current (b) Coil Voltage

Figure 3.10: Measured closed-loop current control response.

6. Generate a Matlab®figure showing the *Implemented Current Control* response and the input voltage.

**Data Saving**: As in s_maglev_pi.mdl, after each run each scope automatically saves their response to a variable in the Matlab®workspace.

7. Measure the steady-state error, the percent overshoot and the peak time of the response. Does the response satisfy the specifications given in Section 3.1.2? **Hint:** Use the Matlab® *ginput* command to take measurements off the figure.
8. To improve the performance of the current control, re-design the PI control gains for a shorter peak time of 0.015 seconds (instead of 0.05 seconds as in Section 3.1.2), i.e., $t_p = 0.015$. Therefore find PI control gains for the following new specifications:
   (a) Natural frequency, $\omega_{n,c} = 350$ rad/s
   (b) Damping ratio, $\zeta = 0.80$
9. Generate a Matlab®figure showing the current control response and the input voltage with the newly designed PI control gains.
10. Measure the steady-state error, the percent overshoot and the peak time of the response. Does the response satisfy the specifications given in Section 3.1.2?

## 3.4 Results

Fill out Table 3.1 with your answers from your control lab results - both simulation and implementation.

| Description | Symbol | Value | Units |
|---|---|---|---|
| **Pre Lab Questions** | | | |
| *Coil Current Control Gains* | | | |
| Proportional Control Gain | $k_{p,c}$ | | V/A |
| Integral Control Gain | $k_{i,c}$ | | V/(A-s) |
| **Current Control Simulation** | | | |
| *1 A Step* | | | |
| Steady-state error | $e_{ss}$ | | deg |
| Peak time | $t_p$ | | s |
| Percent overshoot | PO | | % |
| *2 A Step* | | | |
| Steady-state error | $e_{ss}$ | | deg |
| Peak time | $t_p$ | | s |
| Percent overshoot | PO | | % |
| **Current Control Implementation** | | | |
| *Using PI Gains from Pre-Lab* | | | |
| Steady-state error | $e_{ss}$ | | deg |
| Peak time | $t_p$ | | s |
| Percent overshoot | PO | | % |
| *Re-designed PI Control* | | | |
| Proportional Control Gain | $k_{p,c}$ | | V/A |
| Integral Control Gain | $k_{i,c}$ | | V/(A-s) |
| Steady-state error | $e_{ss}$ | | deg |
| Peak time | $t_p$ | | s |
| Percent overshoot | PO | | % |

Table 3.1: Results

# 4 BALL POSITION CONTROL

## 4.1 Background

### 4.1.1 Specifications

The second and last control strategy is to regulate and track in mid-air the ball position. The closed-loop scheme employed consists of a Proportional-plus-Integral-plus-Velocity (PIV) controller with a feed-forward component.

The first specification is to design the ball position controller for the following operating position (i.e., equilibrium position):

$$x_{b0} = 6 \text{ mm}$$

In response to a desired $\pm 1$ mm square wave position setpoint from the ball equilibrium position in mid-air, the ball position behavior should satisfy the following design performance requirements:

1. Percent Overshoot, less than 5 %, i.e., $PO \leq 5$ %.
2. No static steady-state error, $e_{ss} = 0$.
3. Maximum settling time less than 0.3 second, i.e., $t_{s,b} \leq 0.3$ s.
4. Minimize the control effort produced, which is proportional to the coil input voltage $V_c$. The power amplifier should not go into saturation in any case.

Consider the characteristic equation of a third order transfer function

$$(s + p_0)(s^2 + 2\zeta\omega_n s + \omega_n^2) = s^3 + (p_0 + 2\zeta\omega_n)s^2 + (2p_0\zeta\omega_n + \omega_n^2)s + p_0\omega_n^2. \tag{4.1}$$

In order to achieve the time-domain specifications above, a third-order system with the following parameters is needed:

1. Natural frequency, $\omega_{n,c} = 19.3$ rad/s.
2. Damping ratio, $\zeta = 0.69$.
3. Pole location, $p_0 = 40$ rad/s.

**Settling Time**
The *settling time* of the system is the time it takes for the response to settle within a certain threshold of its final value. Given

$$t_s = t_f - t_0,$$

the parameter $t_f$ is the time it takes for the response to settle within 1%, 2%, or whatever percentage of its final value.

The 2% settling time for a second-order system, i.e., the time required to reach 2% of its final value, can be approximated by

$$t_s = \frac{4}{\zeta\omega_n}. \tag{4.2}$$

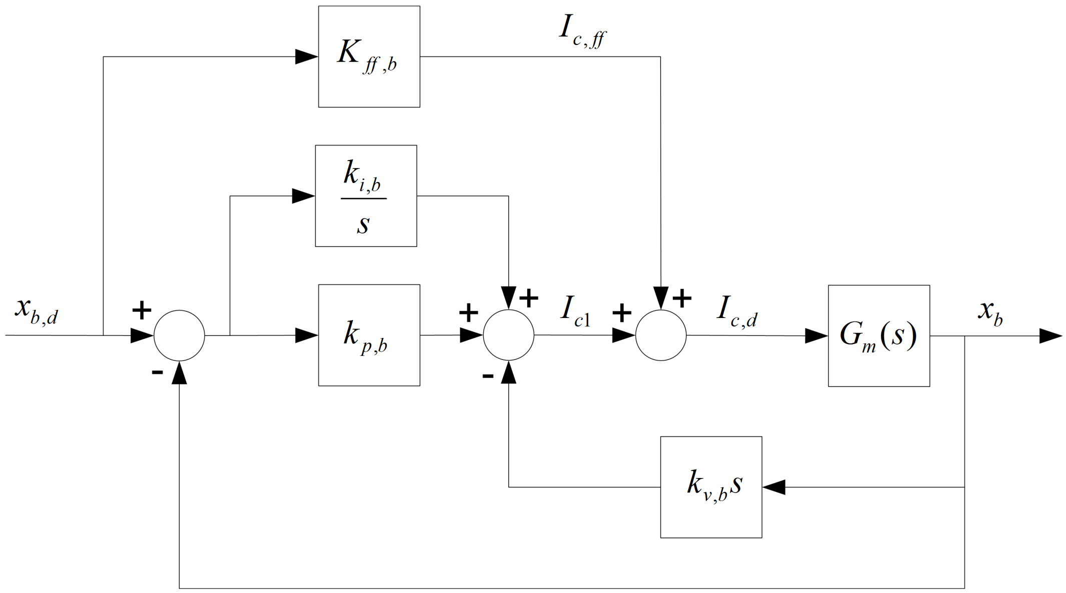



Figure 4.1: Block diagram of PIV+FF ball position control

### 4.1.2 Ball Position Control Design

The steel ball position is controlled using the Proportional-plus-Integral-plus-Velocity (PIV) control and feed-forward action illustrated in Figure 4.1.

As depicted in Figure 4.1, the PIV+FF control is given by

$$I_c(s) = \left(k_{p,b} + \frac{k_{i,b}}{s}\right)(X_{b,d}(s) - X_b(s)) - k_{v,b}sX_b(s) + K_{ff,b}X_{b,d}(s). \tag{4.3}$$

As it can be seen in Figure 4.1, the feed-forward action is necessary since the PIV control system is designed to compensate for small variations (i.e., disturbances) from the linearized operating point $(x_{b0}, I_{c0})$. In other words, while the feed-forward action compensates for the ball gravitational bias, the PIV controller compensates for dynamic disturbances.

The open-loop transfer function $G_m(s)$ takes into account the dynamics of the electromagnet current loop, as characterized in Section 2.1, and is defined

$$G_m(s) = \frac{X_b(s)}{I_{c,d}(s)} = T_c(s)G_b(s)$$

where

$$G_b(s) = \frac{X_b(s)}{I_c(s)}.$$

However to simplify the model, we can neglect the dynamics of the electromagnet current loop. In this analysis of the ball position PIV-plus-feedforward control loop, as presented hereafter, it is therefore assumed that $I_c(s) = I_{c,d}(s)$ and so $T_c(s) = 1$. Given that $G_m(s) = G_b(s)$ and the $G_b(s)$ transfer function found in Section 2.2, we can define the MAGLEV current-to-position model as

$$G_m(s) = \frac{X_b(s)}{I_c(s)} = \frac{-2gx_{b0}}{I_{c0}x_{b0}\left(s^2 - \frac{2g}{x_{b0}}\right)}.$$

Substitute the PIV+FF control in Equation 4.3 into this and solve for $X_b(s)/X_{b,d}(s)$ to obtain the closed-loop ball position transfer function

$$T_b(s) = \frac{X_b(s)}{X_{b,d}(s)} = \frac{2gx_{b0}(K_{ff,b} + k_{p,c})s + k_{i,b}}{-I_{c0}x_{b0}s^3 + 2x_{b0}gk_{v,b}s^2 + (2x_{b0}gk_p + 2I_{c0}g)s + 2x_{b0}gk_i}. \tag{4.4}$$

## 4.2 Pre-Lab Questions

1. Analyze the ball position closed-loop system at the static equilibrium point $(x_{b0}, i_{c0})$ and determine the current feed-forward gain, $K_{ff,b}$. Make sure you evaluate the gain numerically.
2. Find the PIV gains for the ball position control, $k_{p,b}$, $k_{i,b}$, and $k_{v,b}$, in terms of third order system parameters $\omega_n$, $\zeta$, and $p_0$.
3. Based on the MAGLEV operating point parameters, $i_{c0}$ and $x_{b0}$, calculate the control gains needed to satisfy the time-domain response requirements given in Section 4.1.1. iven in Section 4.1.1.

## 4.3 Lab Experiments

### 4.3.1 Ball Position Control Simulation

In this section you will simulate the ball position control of the MAGLEV system. The ball motion and magnet coil dynamics are modeled using the Simulink blocks and controlled using the PIV+FF controller described in Section 4.1.2. Our goals are to confirm that the desired response specifications are satisfied and to verify that the amplifier is not saturated.

**Experimental Setup**

The *s_piv_maglev* Simulink® diagram shown in Figure 4.2 will be used to simulate the closed-loop ball position control response with the PIV+FF ball position controller and PI coil current control used earlier in Section 3.1.3. On the actual device, the ball starts when its on the pedestal at a distance $T_b$. Similarly, in the simulation the ball begins at *Tb*. To prevent a sudden jump, the position setpoint initially starts at *Tb* and gradually commands a step about the operating air gap. The speed of the step is slowed down by a Rate Limiter block.

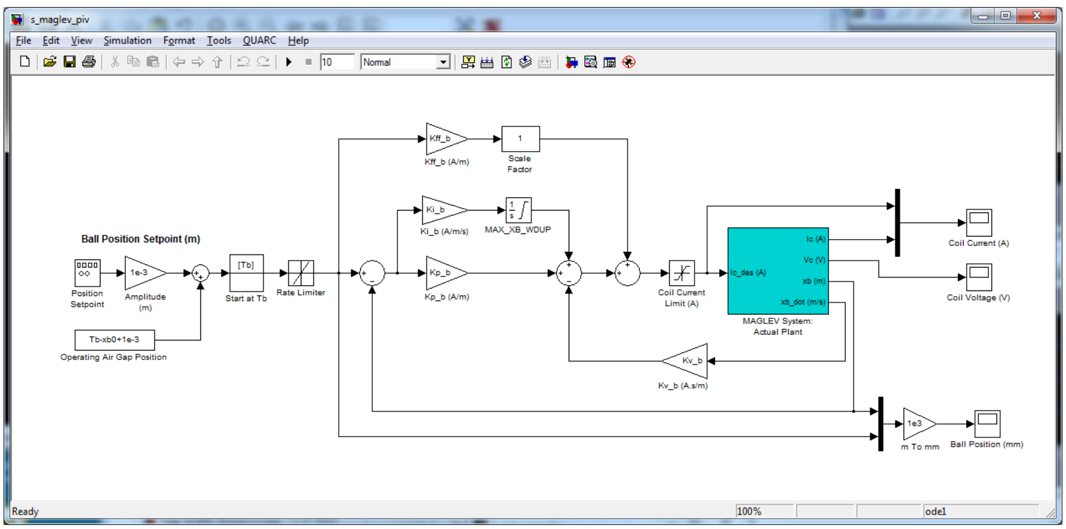
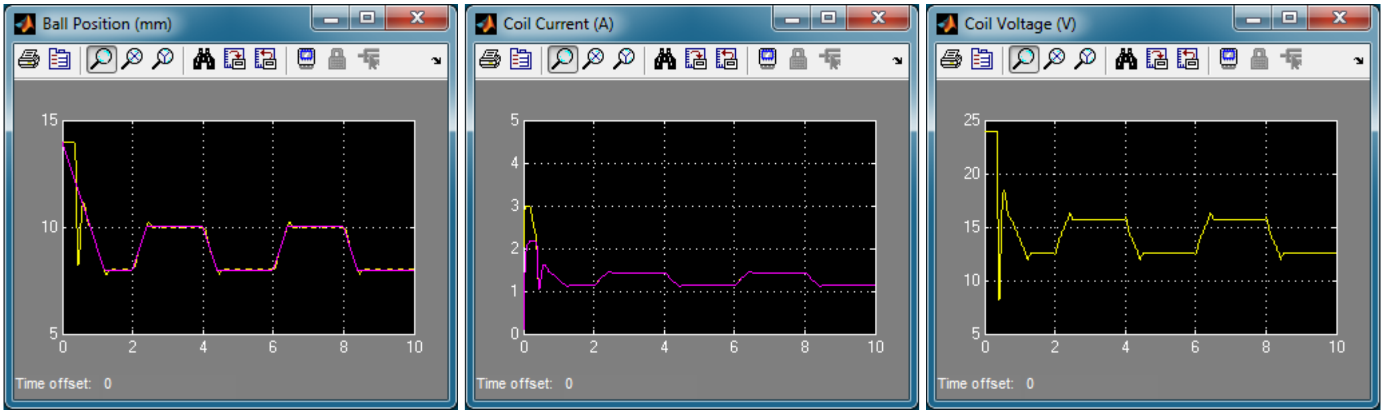



Figure 4.2: Simulink model used to simulate ball position control response.

**IMPORTANT:** Before you can conduct these experiments, you need to make sure that the lab files are configured. If they have not been configured already, then you need to go to Section 5 to configure the lab files first.

1. Enter the current control PI gains in Matlab used in Section 5.2 as *Kp_c* and *Ki_c*.
2. Enter the feed-forward, proportional, integral, and velocity control gains found in Section 4.2 in Matlab® as *Kff_b*, *Kp_b*, *Ki_b*, and *Kv_b*.
3. Set the *Scale Factor* Slider Gain to 1.
4. To generate a step reference, go to the *Position Setpoint* Signal Generator block and set it to the following:
   - Signal type = *square*
   - Amplitude = 1
   - Frequency = 0.25 Hz
5. Set the *Amplitude (m)* gain block to $1e-3$ and the *Operating Air Gap Position* constant block to *-xb0+1e-3* to generate a step that goes between 8 and 10 mm (i.e., $\pm 1$ mm square wave at 0.25 Hz with 9 mm constant).
6. Open the *Ball Position (mm)*, *Coil Current (A)*, and *Coil Voltage (V)* scopes.

7. Start the simulation. By default, the simulation runs for 10 seconds. The scopes should be displaying responses similar to Figure 4.3. Note that in the *Ball Position (m)* and *Coil Current (A)* scopes, the yellow trace is the setpoint (or command) while the purple trace is the simulation.




(a) Ball Position (b) Coil Current (c) Coil Voltage

Figure 4.3: Simulated closed-loop ball position control response.

8. Generate a Matlab®figure showing the *Simulated Ball Position* response, the current, and the input voltage.

   **Data Saving**: Similarly as with s_maglev_pi, after each simulation run each scope automatically saves their response to a variable in the Matlab®workspace. The *Ball Position (mm)* scopes saves its response to the data_xb variable. The *Coil Current (deg)* scope saves its response to the variable called data_Ic and the *Coil Voltage (V)* scope saves its data to the *data_Vc* variable.

9. Measure the steady-state error, the percent overshoot and the peak time of the simulated response. Does the response satisfy the specifications given in Section 4.1.1? Keep in mind, due to the *Rate Limiter* the setpoint is delayed 0.4 seconds. Take that into account. **Hint:** Use the Matlab® *ginput* command to take measurements off the figure.

10. If the simulated response did not satisfy the specifications, explain how you could modify the control system in order for the response to be improved.

### 4.3.2 Ball Position Control Implementation

The q_maglev_piv Simulink diagram shown in Figure 4.4 is used to run the ball position control presented in Section 4.1.2 on the MAGLEV system. The *MAGLEV w/ PI Control* subsystem contains the PI current control used previously in Section 3.3.2 as well as the *MAGLEV* subsystem, which contains QUARC® blocks that interface with the electromagnet and sensors of the MAGLEV system.

**Experimental Setup**

The *q_maglev_piv* Simulink® diagram shown in Figure 4.4 will be used to run the feed-forward and PIV ball position control on the actual MAGLEV system.

**IMPORTANT:** Before you can conduct these experiments, you need to make sure that the lab files are configured according to your setup. If they have not been configured already, then you need to go to Section 5 to configure the lab files first.

Follow this procedure:

1. Enter the current control PI gains in Matlab used in Section 5.2 as *Kp_c* and *Ki_c*.

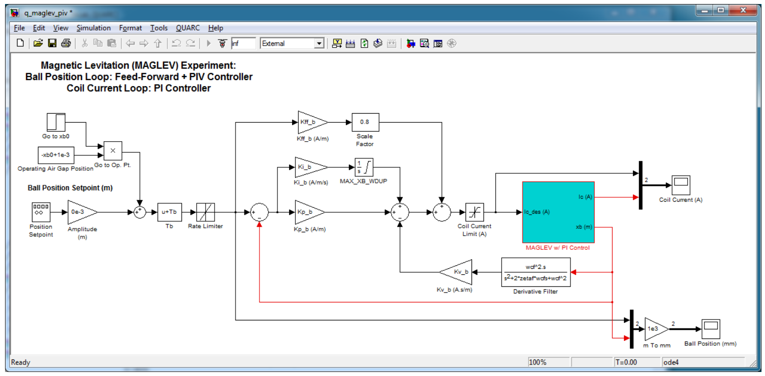
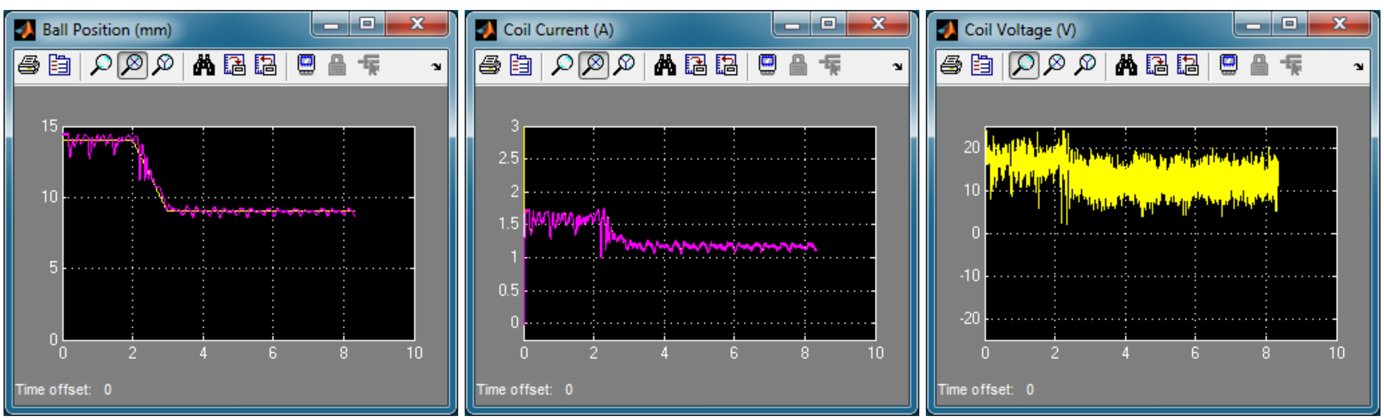



Figure 4.4: Simulink model used to run FF+PIV ball position control on MAGLEV system.

2. Enter the feed-forward, proportional, integral, and velocity control gains found in Section 4.2 in Matlab®as *Kff_b*, *Kp_b*, *Ki_b*, and *Kv_b*.
3. Set the *Scale Factor* Slider Gain block to 0.8.
4. Set the *Amplitude (m)* gain block to 0. This will disable the tracking which makes it easier to stabilize the ball to the initial lift off (i.e., transient period).
5. In the Simulink diagram, go to QUARC | Build.
6. Click on QUARC | Start to run the controller. As the air gap command goes from 14 mm to 9 mm, the ball should eventually rise from its pedestal and stabilize to 9 mm. The scopes should be displaying responses similar to Figure 4.5.




(a) Ball Position (b) Coil Current (c) Coil Voltage

Figure 4.5: Ball position response when stabilizing to 9 mm air gap.

7. Once the ball is stabilized, set the *Amplitude (m)* gain block to 1e-3 meters. This will command a $\pm$ 1 mm square at 0.25 Hz about 9 mm.
8. Generate a Matlab®figure showing the *Implemented Ball Position* response, the current, and the input voltage.

   **Data Saving**: As with s_maglev_piv, after each run the scopes automatically save their response to a variable in the Matlab®workspace. The *Ball Position (mm)* scopes saves its response to the data_xb variable. The *Coil Current (deg)* scope saves its response to the variable called data_Ic and the *Coil Voltage (V)* scope saves its data to the *data_Vc* variable.
9. Measure the steady-state error, the percent overshoot and the peak time of the response obtained on the actual MAGLEV. Does the response satisfy the specifications given in Section 4.1.1? Take into account that the *Rate Limiter* delays the setpoint by 0.4 seconds.

## 4.4 Results

Fill out Table 4.1 with your answers from your control lab results - both simulation and implementation.

| Description | Symbol | Value | Units |
|---|---|---|---|
| **Pre Lab Questions** | | | |
| *Ball Position Control Gains* | | | |
| Feed Forward Control Gain | $K_{ff,b}$ | | A/m |
| Proportional Control Gain | $k_{p,b}$ | | A/m |
| Velocity Control Gain | $k_{v,b}$ | | A-s/m |
| Integral Control Gain | $k_{i,b}$ | | A/(m-s) |
| **Ball Position Control Simulation** | | | |
| Steady-state error | $e_{ss}$ | | deg |
| Settling time | $t_s$ | | s |
| Percent overshoot | PO | | % |
| **Ball Position Control Implementation** | | | |
| Steady-state error | $e_{ss}$ | | deg |
| Settling time | $t_s$ | | s |
| Percent overshoot | PO | | % |

Table 4.1: Results

# 5 SYSTEM REQUIREMENTS

**Required Software**

- Microsoft Visual Studio (MS VS)
- Matlab® with Simulink®, Real-Time Workshop, and the Control System Toolbox
- QUARC®

See the QUARC® software compatibility chart in [3] to see what versions of MS VS and Matlab are compatible with your version of QUARC and for what OS.

**Required Hardware**

- Data acquisition (DAQ) device that is compatible with QUARC®. This includes Quanser DAQ boards such as Q2-USB, Q8-USB, QPID, and QPIDe and some National Instruments DAQ devices. For a full listing of compliant DAQ cards, see Reference [1].
- Quanser Magnetic Levitation.
- Quanser VoltPAQ-X1 power amplifier, or equivalent.

**Before Starting Lab**

Before you begin this laboratory make sure:

- QUARC® is installed on your PC, as described in [2].
- DAQ device has been successfully tested (e.g., using the test software in the Quick Start Guide or the *QUARC Analog Loopback Demo*).
- Magnetic Levitation and amplifier are connected to your DAQ board as described Reference [4].

## 5.1 Overview of Files

| File Name | Description |
|---|---|
| MAGLEV User Manual.pdf | This manual describes the hardware of the MAGLEV system and explains how to setup and wire the system for the experiments. |
| MAGLEV Workbook (Student).pdf | This laboratory guide contains pre-lab questions and lab experiments demonstrating how to design and implement controllers for on the MAGLEV plant using QUARC®. |
| setup_maglev.m | The main Matlab script that sets the MAGLEV control and model parameters. Run this file only to setup the laboratory. |
| config_maglev.m | Returns the MAGLEV system parameters Lc, Rc, Km, Rs, Mb, Tb, g, K_B, and IC_MAX as well as the amplifier limits VMAX_AMP and IMAX_AMP. |
| s_maglev_pi.mdl | Simulink file that simulates magnet coil current controller for the MAGLEV system. |
| s_maglev_piv.mdl | Simulink file that simulates ball position controller for the MAGLEV system. |
| q_maglev_pi.mdl | Simulink file that implements the PI current controller on the MAGLEV system using QUARC®. |
| q_maglev_piv.mdl | Simulink file that implements the PIV ball position controller on the MAGLEV system using QUARC®. |

Table 5.1: Files supplied with the MAGLEV

## 5.2 Setup for Coil Current Control Simulation

Before beginning the in-lab procedure outlined in Section 3.3.1, the s_maglev_pi Simulink diagram and the setup_maglev.m script must be configured.

Follow these steps:

1. Load the Matlab® software.
2. Browse through the *Current Directory* window in Matlab and find the folder that contains the file *setup_maglev.m*.
3. Open the *setup_maglev.m* script.
4. **Configure setup_maglev.mscript**: Make sure the script is setup to match your system configuration:
   - K_AMP to 3 (unless your amplifier gain is different)
   - AMP_TYPE to your amplifier type (e.g., VoltPAQ).
   - CONTROLLER_TYPE to 'MANUAL'.
5. Run setup_maglev.m to setup the Matlab workspace.
6. Open the *s_maglev_pi.mdl* Simulink diagram, shown in Figure 3.7.

## 5.3 Setup for Ball Position Control Simulation

Before beginning the in-lab procedure outlined in Section 4.3.1, the s_maglev_piv Simulink diagram and the setup_maglev.m script must be configured.

Follow these steps:

1. Go through the steps outlined in Section 5.2 to configure the setup_maglev.m script properly.
2. Run setup_maglev.m to setup the Matlab workspace.
3. Open the *s_maglev_piv.mdl* Simulink diagram, shown in Figure 4.2.

## 5.4 Setup for Implementing Coil Current Control

Before performing the in-lab exercises in Section 3.3.2, the q_maglev_pi Simulink diagram and the setup_maglev.m script must be configured.

Follow these steps to get the system ready for this lab:

1. Setup the MAGLEV as detailed in the MAGLEV User Manual ([4]).
2. If using the VoltPAQ-X1, **make sure the Gain switch is set to 3**.
3. Place the metal ball on the MAGLEV pedestal.
4. Configure and run setup_maglev.m as explained in Section 5.2.
5. Open the *q_maglev_pi.mdl* Simulink diagram, shown in Figure 3.9.
6. **Configure DAQ:** Ensure the HIL Initialize block in the *MAGLEV* subsystem is configured for the DAQ device that is installed in your system. See Reference [1] for more information on configuring the HIL Initialize block.

## 5.5 Setup for Implementing Ball Position Control

Before performing the in-lab exercises in Section 4.3.2, the q_maglev_piv Simulink diagram and the setup_maglev.m script must be configured.

Follow these steps to get the system ready for this lab:

1. Follow steps 1-4 given in the Current Control Implementation setup in Section 5.4.
2. Open the *q_maglev_piv.mdl* Simulink diagram, shown in Figure 4.4.
3. **Configure DAQ:** Ensure the HIL Initialize block in the *MAGLEV* subsystem is configured for the DAQ device that is installed in your system. See Reference [1] for more information on configuring the HIL Initialize block.

# 6 LAB REPORT

This laboratory contains two groups of experiments, namely,

1. Coil current control, and
2. Ball position control.

For each experiment, follow the outline corresponding to that experiment to build the *content* of your report. Also, in Section 6.3 you can find some basic tips for the *format* of your report.

## 6.1 Template for Coil Current Control Report

**I. PROCEDURE**

1. *Simulation*
   - Briefly describe the main goal of the simulation.
   - Briefly describe the simulation procedure in Step 8 in Section 3.3.1.
   - Briefly describe your proposed solution in Step 10 in Section 3.3.1.
   - Briefly describe the changes to the control in Step 11 in Section 3.3.1.
2. *Implementation*
   - Briefly describe the main goal of this experiment.
   - Briefly describe the experimental procedure in Step 6 in Section 3.3.2.

**II. RESULTS**
Do not interpret or analyze the data in this section. Just provide the results.

1. Response plot from step 8 in Section 3.3.1, *Current control simulation*.
2. Controller modifications in step 11 in Section 3.3.1.
3. Response plot from step 12 in Section 3.3.1 using the modified PI control.
4. Response plot from step 6 in Section 3.3.2, *Current control implementation*.
5. Response plot from step 9 in Section 3.3.2, *Tuned current control implementation*.
6. Provide applicable data collected in this laboratory (from Table 3.1).

**III. ANALYSIS**
Provide details of your calculations (methods used) for analysis for each of the following:

1. Peak time, percent overshoot, steady-state error, and input voltage in Step 9 in Section 3.3.1.
2. Effect of increasing current setpoint step to 2 A in Step 10 in Section 3.3.1.
3. Peak time, percent overshoot, steady-state error, and input voltage in Step 13 in Section 3.3.1.
4. Peak time, percent overshoot, steady-state error, and input voltage in Step 7 in Section 3.3.2.

**IV. CONCLUSIONS**
Interpret your results to arrive at logical conclusions for the following:

1. Whether the controller meets the specifications in Step 9 in Section 3.3.1, *Current controller simulation*.
2. Whether the controller meets the specifications in Step 13 in Section 3.3.1, *Re-designed current controller simulation*.
3. Whether the controller meets the specifications in Step 7 in Section 3.3.2, *Current controller implementation*.
4. Whether the controller meets the specifications in Step 10 in Section 3.3.2, *Tuned current controller implementation*.

## 6.2 Template for Ball Position Control Report

**I. PROCEDURE**

1. *Simulation*
   - Briefly describe the main goal of the simulation.
   - Briefly describe the simulation procedure in Step 8 in Section 4.3.1.
   - Briefly describe the control modification procedure in Step 10 in Section 4.3.1.
2. *Implementation*
   - Briefly describe the main goal of this experiment.
   - Briefly describe the experimental procedure in Step 8 in Section 4.3.2.

**II. RESULTS**
Do not interpret or analyze the data in this section. Just provide the results.

1. Response plot from step 8 in Section 4.3.1, *Ball position control simulation*.
2. Response plot from step 8 in Section 4.3.2, *Ball position control implementation*.
3. Provide applicable data collected in this laboratory (from Table 4.1).

**III. ANALYSIS**
Provide details of your calculations (methods used) for analysis for each of the following:

1. Peak time, percent overshoot, steady-state error, and input voltage in Step 9 in Section 4.3.1.
2. Peak time, percent overshoot, steady-state error, and input voltage in Step 9 in Section 4.3.2.

**IV. CONCLUSIONS**
Interpret your results to arrive at logical conclusions for the following:

1. Whether the controller meets the specifications in Step 9 in Section 4.3.2, *Ball position controller implementation*.

## 6.3 Tips for Report Format

**PROFESSIONAL APPEARANCE**

- Has cover page with all necessary details (title, course, student name(s), etc.)
- Each of the required sections is completed (Procedure, Results, Analysis and Conclusions).
- Typed.
- All grammar/spelling correct.
- Report layout is neat.
- Does not exceed specified maximum page limit, if any.
- Pages are numbered.
- Equations are consecutively numbered.
- Figures are numbered, axes have labels, each figure has a descriptive caption.
- Tables are numbered, they include labels, each table has a descriptive caption.
- Data are presented in a useful format (graphs, numerical, table, charts, diagrams).
- No hand drawn sketches/diagrams.
- References are cited using correct format.

# REFERENCES


[1] Quanser Inc. *QUARC User Manual*.

[2] Quanser Inc. *QUARC Installation Guide*, 2009.

[3] Quanser Inc. *QUARC Compatibility Table*, 2010.

[4] Quanser Inc. *Magnetic Levitation User Manual*, 2012.

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